2026 TUESDAY PROBLEMS #5

9 november

1. Show that any function can be written as the sum of an even function and an odd function.

Show that any square matrix can be written as a sum of a symmetric matrix and a skew-symmetric matrix.

(Recall that $f(x)$ is *even* if $f(x) = f(-x)$ for all x and $f(x)$ is *odd* if $f(x) = -f(-x)$ for all x.)

2. Evaluate $\lim_{n\to\infty} \sum_{j=1}^{n} \frac{n}{n^2+j^2}$.

3. Show that for any integers a, b there is an integer c with $a(a+b)(a+2b)(a+3b)+b^4 = c^2$.

4. Find all prime numbers p such that $2^p + p^2$ is also prime.

5. Let (x_1, y_1), (x_2, y_2) and (x_3, y_3) be three distinct points on the parabola $y = ax^2$, such that their normal lines intersect in a common point. Prove that $x_1 + x_2 + x_3 = 0$.