

2025 TUESDAY PROBLEMS #8

29 october

1. For which positive a, b is $a^{\ln(b)} = b^{\ln(a)}$?
2. The Fibonacci numbers are defined by $f_1 = f_2 = 1$ and $f_{n+2} = f_{n+1} + f_n$.
Express $f_1 + f_2 + \cdots + f_n$ in terms of a single Fibonacci number.
3. Show that if $2^n - 1$ is prime, then n is prime.
4. Choose any 1000 (distinct) points in the plane. Does there always exist a straight line which divides the points exactly in two, so that exactly 500 points are on each side of the line and exactly 500 are on the other side?
[A SIDE of a line is one of the two connected components that remain when the line is deleted from the plane.]
5. Consider a sequence of real numbers with the property that the sum of any 7 consecutive terms is positive and the sum of any 11 consecutive terms is negative. Show that such a sequence must be finite, and find the maximum length possible. What if 7 and 11 are replaced by arbitrary (distinct) positive integers?