2025 TUESDAY PROBLEMS #4

3 april

1. Prove that if the real numbers a, b, c satisfy the following, then one of a, b, c is the sum of the other two.

$$|a-b| \geq |c|, \qquad |b-c| \geq |a|, \qquad |c-a| \geq |b|$$

2. Find the positive solutions of the system of equations:

$$x_1 + \frac{1}{x_2} = 4, \quad x_2 + \frac{1}{x_3} = 1, \quad x_3 + \frac{1}{x_4} = 4, \quad \cdots, \quad x_{99} + \frac{1}{x_{100}} = 4, \quad x_{100} + \frac{1}{x_1} = 1,$$

3. Choose n points on a circle and draw all $\binom{n}{2}$ chords defined by these points. Suppose that no three of these chords have a common intersection point. How many regions do these chords divide the circle into?

4. Four points are chosen at random on the surface of a sphere. What is the probability that the centre of the sphere lies inside the tetrahedron whose vertices are the four points?

5. Consider a polyhedron with at least five faces such that exactly three edges emerge from each of its vertices. Two players play the following game.

> Each player, in turn, signs their name on a previously unsigned face. The winner is the player who fiurst succeeds in signing three faces that share a common vertex.

Show that the player who signs first will always win by playing as well as possible.