

2025 TUESDAY PROBLEMS #3

20 march

1. Find the maximum value of $f(x) = x^3 - 3x$ on the set of all real numbers x satisfying $x^4 + 36 \leq 13x^2$.

2. If $a, b, c > 0$, is it possible that each of the following polynomials has two real roots?

$$p(x) = ax^2 + bx + c$$

$$q(x) = bx^2 + cx + a$$

$$r(x) = cx^2 + ax + b$$

3. Find the smallest positive integer n such that for every integer m with $0 < m < 1993$, there exists an integer k for which

$$\frac{m}{1993} < \frac{k}{n} < \frac{m+1}{1994}$$

4. Shanille O'Keal shoots free throws on a basketball court. She hits the first and misses the second, and thereafter the probability that she hits the next shot is equal to the proportion of shots she has hit so far. What is the probability she hits exactly 50 of her first 100 shots?

5. A drunken person stands one step away from a cliff. They take random steps, either directly away from the cliff with probability p or directly towards the cliff with probability $1 - p$. Find the probability of survival, as a function of p .