2025 TUESDAY PROBLEMS #2

10 march

1. Let $p(x)$ be a polynomial with integer coefficients such that $p(0)$ and $p(1)$ are both odd. Show that p has no integer zeros.

2. Show that for $n \geq 2$, $(1 + x + \cdots + x^n)^2 - x^n$ is the product of two polynomials of positive degree with integer coefficients.

3. Let $f(x)$ be a polynomial with real coefficients, and suppose that $f(x) + f'(x) > 0$ for all x. Prove that $f(x) > 0$ for all x.

4. What is the maximum nbumber of rational points that can lie on a circle in R^2 whose centre is not a rational point? (A RATIONAL POINT is a point both of whose coordinates are rational numbers.)

5. Let f be a nonconstant polynomial with positive integer coeffiients. Prove that if n is a positive integer, then $f(n)$ divides $f(f(n) + 1)$ if and only if $n = 1$