2025 TUESDAY PROBLEMS #1

3 march

1. Find the maximum of $\frac{2019^n}{n!}$ over all positive integers n.

2. Let n be an even positive integer, and let $p(x)$ be a polynomial of degree n such that $p(k) = p(-k)$ for $k = 1, 2, \cdots, n$. Prove that there is a polynomial $q(x)$ such that $p(x) = q(x^2)$.

3. Let $a_n = 10 + n^2$ for $n \geq 1$. For eacn $n \geq 1$, let $d_n = \gcd(a_n, a_{n+1})$. Find the maximum value of d_n.

4. Determine all positive integers n for which there exist positive integers a, b, c satisfying $2a^n + 3b^n = 4c^n$.

5. Let n and k be positive integers. The square in the i-th row and j-th column of an $n \times n$ grid contains the number $i + j - k$. For which n and k is it possible to select n squares from the grid, no two in the same row or column, such that the numbers contained in the selected squares are exactly $1, 2, \cdots, n$?