2025 TUESDAY PROBLEMS #11

3 december : Saturday is Putnam...

1. Show that any convex polyhedron has two faces with the same number of edges.

2. Let N be a positive integer. Prove that there is an integer multiple of N whose decimal representation contains all ten decimal digits.

3. Show that the sum $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ is never an integer for $n \geq 2$.

4. Prove that there are no primes in the folowing infinite sequence of integers.

$$1001, 1001001, 1001001001, 1001001001001, \cdots$$

5. Let $f : [0, 1] \to \mathbb{R}$ be continuous, with $f(0) = f(1)$.
Show that for some $x \in [0, 1998/1999]$ with $f(x) = f(x + 1/1999)$.