

# 2025 TUESDAY PROBLEMS #10

26 november

hints, one per page

close your eyes and press “nextpage”  $n$  times for a hint to problem  $n$

1. trig identity?

**2.** What is the sum of the rightmost digit in all the numbers from one to one billion?

- 3.** A good start would be to figure out the eigenvectors. Specifically, can you express the eigenvectors for  $A_j$  in terms of the eigenvectors for  $A_{j-1}$ ?

4. You should be able to convince yourself that if  $2\pi/\theta$  is an integer, that it works.

Pick some  $\theta$ , and cut the cake. Specifically, look at exactly what happens to the  $(k+1)$ -th piece, where  $k$  is the (unique) integer with  $k\theta < 2\pi < (k+1)\theta$ . What, exactly, does happen? And what happens after  $4\pi$ ?

